COMPUTER PROGRAMS FOR PREDICTION OF STRUCTURAL VIBRATIONS DUE TO FLUCTUATING PRESSURE ENVIRONMENTS

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Formulations were derived and computer programs were written to calculate the random vibrational responses of rectangular cylindrical panels cross-reinforced with ribs and stringers subjected to the fluctuating pressure environments. Three boundary conditions are considered: four edges simply supported; four edges clamped; two opposite edges simply supported and the other two clamped. Special cases of complete cylinders and flat panels are included. Either the spectral density or the one-third-octave level of the excitation pressure may be input in any discrete frequencies. Formulations are according to the normal mode approach. The responses calculated are the acceleration, displacement, and stress spectral densities, overall mean-square and root-mean-square values. All spectral densities are tabulated and plotted. New expressions for the joint acceptance of all mode combinations for different correlation functions are derived. Both local responses at any point and the average responses over the complete panel are calculated. Comparison of calculated results with test data shows good agreement.

INTRODUCTION

The purpose of this project was to develop computer programs to calculate the random vibrational responses of rectangular cylindrical shell panels cross-reinforced with ribs and stringers. The boundary conditions considered are four edges simply-supported, four edges clamped, and two opposite edges simply-supported while other two clamped. Special cases of complete cylinders and flat panels are included.

A total of eleven computer programs have been developed. The main program RANDOM contains the formulations for the three boundary conditions and will calculate the responses for any one of the boundary conditions. For accurate results and large frequency range, programs were written for each boundary condition to perform special investigations.

The one-third-octave spectrum of the excitation pressure is input in any discrete frequencies. Any excitation pressure spectrum of any shape can be input. This improves the simulation of the excitation pressure field. The excitation spectrum is converted into pounds-per-square-inch squared per Hertz. Excitation pressure for each data point frequency is obtained by interpolation. The excitation pressure spectrum is plotted both in decibel scale and in $(psi)^2$ per Hertz.

The formulations are according to the normal mode approach. Both Alan Powell's [1] joint acceptance and Y.K. Lin's [2] cross spectral density of the generalized force are used in the formulations. The relation of these two quantities is given. The analytical expressions of these two quantities for all mode combinations for two correlation functions are derived. One correlation function is exponentially decaying with separation distance and frequency while the other is a cosine function with exponentially decaying amplitude. Separate computer programs are written to study the joint acceptance and the cross spectral density of the generalized force.

Contributions of both main terms and cross terms are summed to obtain the responses. The one-nth-octave bandwidth is used for frequency increment to save computer time and yield smooth response spectral density plots. The frequency range for the spectrum is 5000 Hertz or more. Up to 625 terms are summed to give the response spectral density at each data point. More than 1000 date points can be calculated for each response spectral density plot. The responses are calculated as the displacement, the acceleration, and the stress spectral densities. Mean-square and root-mean-square values are calculated by numerical integration. Response spectral densities are tabulated and plotted with the root-mean-square value printed at the top of the plot. The programs will apply when either the complete panel or a portion is exposed to the excitation pressure. Both local responses at any point and average responses over the whole panel can be calculated.

In one of the programs, the acceleration spectral density is expressed in decibels referenced gravity acceleration and the vibro-acoustic transfer function is calculated as the acceleration spectral density minus the excitation. This is useful for the investigation of the transfer function of structures.

Separate programs are written to investigate the contribution of cross terms of the total response. It is found that the cross terms though do not contribute very much to the mean-square response, they do affect the shape of the response spectral density to a certain degree.

Natural frequencies of the panel are calculated in the programs. These frequency equations are newly derived or modification of those available in the literature. For panels of uniform thickness, these equations are the same as those in the literature. For cylindrical shell panels cross-reinforced with stiffeners, no frequency equations can be found in the literature that can be advantageously used in the programs. These newly derived equations though approximate in nature, yet yield reasonable results. The frequency equations account for the boundary conditions, the rigidity of the stiffeners, and the curvature of the shell. They are not very complicated so they can be incorporated into the computer programs without requiring a large amount of computer time.

A program is written to calculate the modal density and the total number of natural frequencies up to a certain range. This program is useful in the investigation of dynamic characteristics of structures.

By utilization of the developed computer programs, an investigation on the effect of boundary conditions on the responses is performed. It is found that the more rigid the boundary condition, the less the displacement spectral density and the larger the acceleration spectral density at resonance. The root-mean-square displacements for the three boundary conditions are not much different. Estimations of responses are made by the spectral density at fundamental mode. It is found that the fundamental mode contributes up to 50% of the mean-square responses, and the second mode contributes more to the acceleration than the displacement response.

Comparison of computer results with experimental data from project, the Chrysler Huntsville Operations was conducting for Marshall Space Flight Center, shows good agreement.

Only important formulations are given here and details of the equations will be found in the given references.

NOMENCLATURE

A_1	Decay constant along x-axis
A_2	Decay constant along y-axis
A_{jkmn}	See eq. (15)
B_{jkmn}	See eq. (15)
C_{jkmn}	See eq. (15)
D_x	Rigidity, eq. (2)
D_y	Rigidity, eq. (2)
E	Young's modulus of panel skin
E'	Young's modulus of stiffeners
$F_{jk}(\vec{r})$	Normal mode
H	Rigidity, eq. (2)
H_{jk}	Frequency response function
H^*_{mn}	Conjugate of H_{mn}
I_1	Moment of inertia of one length-direction stiffener with respect to neutral axis
I_2	Moment of inertia of one width-direction stiffener with respect to neutral axis
I_x, I_y	Integrals, eq. (33)
$I_{jm}, I'_{jm}, I_{kn}, I'_{kn}$	See equation (10)
I_{jkmn}	Cross spectral density of generalized force
$\tilde{I}_{jkmn}$	Normalized cross spectral density of generalized force
J_{jkmn}	Joint acceptance
$K = \frac{\omega}{c}$	Wave number
L_a	Excitation overall pressure level in db
M	Smeared-out mass per unit area
M_{jk}	Modal mass
$\Omega_x, \Omega_y, \Omega_w$	See eqs. (30), (31) and (32)
S	Area of panel
$S' = b'\ell'$	Area of panel subjected to excitation
$S_{pp}(f)$	Excitation spectral density in db/Hz
$S'_{pp}(f)$	Excitation spectral density in psi^2/Hz
$S_{\ddot{w}g}(\vec{r},f)$	Acceleration spectral density in db referenced g
$S_{ww}(\vec{r},f)$	Displacement spectral density in in^2/Hz
$S_{\ddot{w}\ddot{w}}(\vec{r},f)$	Acceleration spectral density in g^2/Hz
$S_{\sigma\sigma}(\vec{r},f)$	Stress spectral density in psi^2/Hz
$S_{3ro}(f)$	One-third-octave excitation pressure level in db
$T_{\ddot{w}}(\vec{r},f)$	Vibro-acoustic transfer function in db
$X_j(x)$	See eq. (7)
$Y_k(y)$	See eq. (7)
a	Radius of shell
a_1	Spacing of width-direction stiffeners
b	Circumferential width of panel
b'	Width of panel subjected to excitation
b_1	Spacing of length-direction stiffeners
c	Speed of sound in medium
f	Frequency of Hertz
g	Gravity acceleration
h	Thickness of panel skin
h'	Smeared-out thickness of stiffeners
$h_1 = h + h_2$	Height
h_2	Largest height of stiffeners at $\vec{r}$ (see figure 1)
j,k,m,n	Mode indices

ℓ	Axial length of panel
ℓ'	Length of panel subjected to excitation
n	One-nth-octave increment
p_a^2	Overall mean-square pressure in psi^2
$\vec{r}$	Position vector
$w(\vec{r})$	Root-mean-square displacement
$w^2(\vec{r})$	Mean-square displacement
x,y	Cartesian coordinates of $\vec{r}$
$\tilde{x}_j$	See eq. (8)
$\Phi_{pp}(\vec{r}_1,\vec{r}_2,\omega)$	Correlation function
$\Phi_{pp}(\omega)$	Excitation spectral density in psi^2/rad/sec
$\Phi_{ww}(\vec{r},\omega)$	Displacement spectral density in in^2/rad/sec
$\Phi_{\ddot{w}\ddot{w}}(\vec{r},\omega)$	Acceleration spectral density in in^2/sec^4/ rad/sec
$\Phi_{\sigma\sigma}(\vec{r},\omega)$	Stress spectral density in psi^2/rad/sec
α_1, α_2	See eq. (12)
$\gamma^2(\vec{r})$	Constant to convert displacement spectral density into stress
ζ_{jk}	Damping ratio
η	Separation distance along y-axis
ν	Poisson's ratio
ξ	Separation distance along x-axis
ρ	Mass density of panel skin
ρ'	Mass density of stiffeners
$\rho(\vec{r}_1,\vec{r}_2)$	Correlation coefficient
ω	Frequency in rad/sec
ω_{jk}	Natural frequency in rad/sec

FREQUENCY EQUATIONS

The frequency equations are newly derived and modification of those available in the literature. The expressions of frequency equations are different for lower modes and higher modes and boundary conditions. Detail expressions and derivation of the frequency equations are given in[3]. Typical one for higher modes of two opposite edges simply-supported and other two clamped panel is as follows:

$$\omega_{jk} = \frac{\pi^2}{\sqrt{M}}\left\{D_x\left(\frac{j}{\ell}\right)^4 + D_y\left(\frac{k+1/2}{b}\right)^4 + 2H\left(\frac{j}{\ell}\right)^2 \frac{(k+1/2)\left[(k+1/2) - 2/\pi\right]}{b^2} + \frac{Eh}{a^2\pi^4\left[1+\left(\frac{j}{k+1/2}\right)^2\left(\frac{\ell}{b}\right)^2\right]^2}\right\}^{1/2} \quad (1)$$

$j,k = 2,3, \ldots$

Refer to Figure 1 for geometric dimensions:

$$D_x = \frac{Eh^3}{12(1-\nu^2)} + \frac{E'I_1}{b_1}$$

$$D_y = \frac{Eh^3}{12(1-\nu^2)} + \frac{E'I_2}{a_1}$$

$$H = \frac{Eh}{12(1-\nu^2)} \quad (2)$$

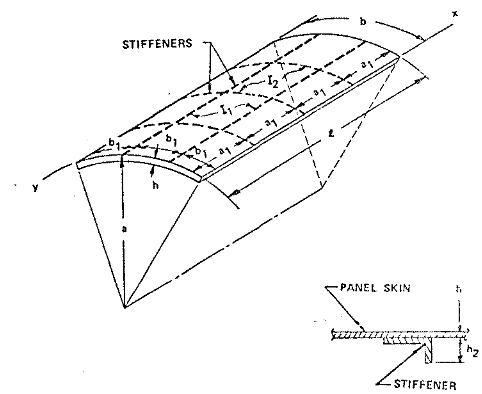


FIGURE 1. GEOMETRY OF RECTANGULAR CYLINDRICAL SHELL PANEL CROSS–REINFORCED WITH STIFFENERS

It is seen that the effects of the stiffeners are accounted for by the quantities D_x and D_y, and the influence of the radius is expressed by the last term in the equation. When the rigidities of the stiffeners approach zero, the frequency equation reduces to that of unstiffened panel. As the radius approaches infinity, the frequency equation is that of flat panels.

CORRELATION FUNCTIONS

The correlation of the excitation pressure field is represented by the following two correlation coefficients:

$$\rho_1(\xi) = \exp(-A_1K\xi) \quad (3)$$

$$\rho_2(\xi) = \exp(-A_2K\xi)\cos(K\xi) \quad (4)$$

For spatial homogeneous pressure field, the cross spectral density functions of the excitation corresponding to the above correlation will be:

$$\Phi_{pp}(\vec{r}_1,\vec{r}_2,\omega) = \Phi_{pp}(\omega)\exp(-A_1K\xi)\exp(-A_2K\eta) \quad (5)$$

$$\Phi_{pp}(\vec{r}_1,\vec{r}_2,\omega) =$$

$$\Phi_{pp}(\omega) \exp(-A_1 K \xi) \cos(K\xi) \exp(-A_2 K \eta) \cos(K\eta) \qquad (6)$$

where

$$\xi = |x_1 - x_2|$$

$$\eta = |y_1 - y_2|$$

NORMAL MODES

Normal modes are represented by

$$F_{jk}(\vec{r}) = X_j(x) Y_k(y) \qquad (7)$$

where X_j and Y_k are functions of x and y respectively. Expressions for these functions for different boundary conditions are given in[3]. Following is a typical one for the higher modes of the four edges clamped panel.

$$X_j(x) = \cosh \tilde{x}_j - \cos \tilde{x}_j - \sinh \tilde{x}_j + \sin \tilde{x}_j$$

$$j = 2, 3 \ldots \qquad (8)$$

$$\tilde{x}_j = \frac{(j + 1/2)\pi x}{\ell}$$

JOINT ACCEPTANCE AND NORMALIZED CROSS SPECTRAL DENSITY OF GENERALIZED FORCE

Analytical expressions for the joint acceptance and the cross spectral density of the generalized force are derived for all mode combinations for two different correlation functions. Formulations for numerical calculation of these quantities are developed for any correlation function and mode shape. The cross spectral density of the generalized force is defined[2].

$$I_{jkmn} = \iint \Phi_{pp}(\vec{r}_1,\vec{r}_2,\omega)\, F_{jk}(\vec{r}_1)\, F_{mn}(\vec{r}_2)\, d\vec{r}_1\, d\vec{r}_2 \qquad (9)$$

For the four edges simply-supported panel and the excitation cross spectral density functions of eqs. (5) and (6), the expressions for I_{jkmn} have been obtained in closed forms. For the latter spectral density function, the complete expressions cover nine pages and can be found in[3]. The expressions for the former density function are as follows.

$$I_{jkmn} = (b\ell)^2\, \Phi_{pp}(\omega)\, I_{jm} I_{kn} \quad \text{for } j = m,\ k = n$$

$$= (b\ell)^2\, \Phi_{pp}(\omega)\, I_{jm} I'_{kn} \quad \text{for } j = m,\ k \neq n$$

$$= (b\ell)^2\, \Phi_{pp}(\omega)\, I'_{jm} I_{kn} \quad \text{for } j \neq m,\ k = n$$

$$= (b\ell)^2\, \Phi_{pp}(\omega)\, I'_{jm} I'_{kn} \quad \text{for } j \neq m,\ k \neq n \qquad (10)$$

where

$$I_{kn} = \frac{\alpha_2{}^2}{2}\left[\frac{1}{\alpha_2{}^2 + (k\pi)^2} + \frac{1}{\alpha_2{}^2 + (n\pi)^2}\right]$$

$$+ (k\pi)(n\pi)\frac{2 + \left[(-1)^{k+1} + (-1)^{n+1}\right]\exp(-\alpha_2)}{\left[\alpha_2{}^2 + (k\pi)^2\right]\left[\alpha_2{}^2 + (n\pi)^2\right]} \quad \text{for } k = n$$

$$I'_{kn} = (k\pi)(n\pi)\frac{2 + \left[(-1)^{k+1} + (-1)^{n+1}\right]\exp(-\alpha_2)}{\left[\alpha_2{}^2 + (k\pi)^2\right]\left[\alpha_2{}^2 + (n\pi)^2\right]}$$

$$+ \frac{k}{\alpha_2{}^2 + (k\pi)^2}\left[\frac{(-1)^{n-k} - 1}{2(n-k)} + \frac{(-1)^{n+k} - 1}{2(n+k)}\right]$$

$$+ \frac{n}{\alpha_2{}^2 + (n\pi)^2}\left[\frac{(-1)^{k-n} - 1}{2(k-n)} + \frac{(-1)^{k+n} - 1}{2(k+n)}\right] \quad \text{for } k \neq n \qquad (11)$$

$$\alpha_1 = A_1 K \ell \qquad \alpha_2 = A_2 K b \qquad K = \frac{\omega}{c} \qquad (12)$$

Replacing k by j, n by m, and α_2 by α_1 in the above expressions gives I_{jm} and I'_{jm}.

The normalized cross spectral density of the generalized force of the excitation is defined as

$$\tilde{I}_{jkmn} = \frac{I_{jkmn}}{S^2\, \Phi_{pp}(\omega)} \qquad (13)$$

where the area of the panel is $S = b\,\ell$

The relation between the joint acceptance and the normalized cross spectral density of the generalized force is given by

$$J_{jkmn} = \tilde{I}_{jkmn} \cos \omega(\tau_{jk} - \tau_{mn}) \qquad (14)$$

where

$$\cos \omega(\tau_{jk} - \tau_{mn}) = \frac{A_{jkmn}}{C_{jkmn}}$$

$$A_{jkmn} = \left[1 - (\omega/\omega_{jk})^2\right]\left[1 - (\omega/\omega_{mn})^2\right] + 4\zeta_{jk}\zeta_{mn}\, \omega^2/(\omega_{jk}\omega_{mn})$$

$$C^2_{jkmn} = A^2_{jkmn} + B^2_{jkmn}$$

$$B_{jkmn} = 2\left\{\left[1 - (\omega/\omega_{jk})^2\right]\zeta_{mn}\omega/\omega_{mn} - \left[1 - (\omega/\omega_{mn})^2\right]\zeta_{jk}\omega/\omega_{jk}\right\} \qquad (15)$$

It is seen that $\tilde{I}_{jkjk}$ and J_{jkjk} are identical. Computer Program JARSR1 was written to calculate $\tilde{I}_{jkmn}$ and J_{jkmn}. Figures 2, 3 and 4 are typical plots.

EXCITATION PRESSURE DATA

The excitation spectral density in psi^2/rad/sec is given by

$$\Phi_{pp}(\omega) = \frac{1}{2\pi}[10]^{\frac{S_{pp}(f) - 170.576}{10}} \qquad (16)$$

The excitation spectral density in decibels/Hertz is given by

$$S_{pp}(f) = S_{3rd}(f) - 10 \log_{10}(0.23157 f) \qquad (17)$$

where $S_{3rd}(f)$ is the one-third octave excitation pressure level in decibels.

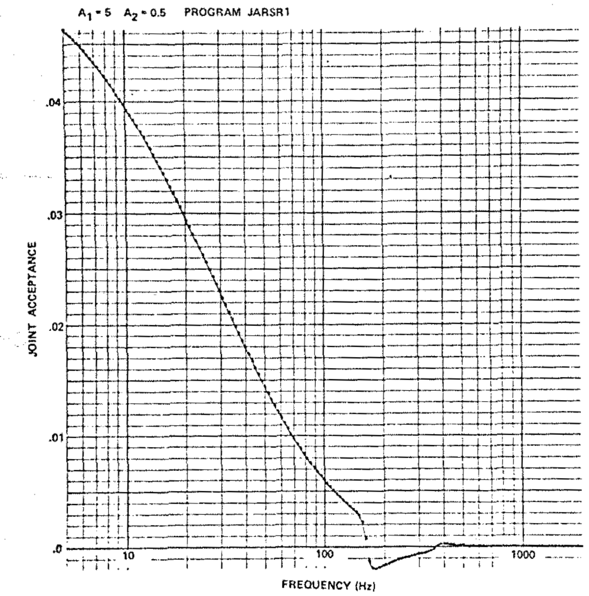


FIGURE 2. JOINT ACCEPTANCE, EXPONENTIALLY DECAYING CORRELATION FUNCTION, MODE INDICES: 1, 1, 1, 3

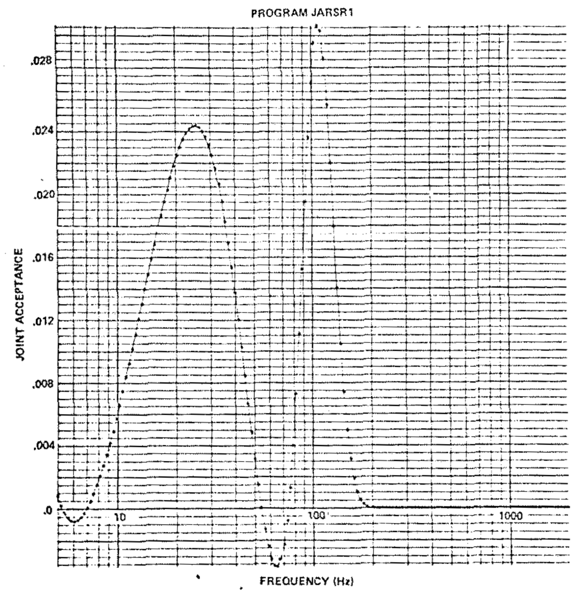


FIGURE 3. JOINT ACCEPTANCE, SINUSOIDAL DECAYING CORRELATION FUNCTION MODE INDICES: 1, 1, 1, 1

The excitation spectral density in $(\text{psi})^2$/Hertz is given by

$$S'_{pp}(f) = 2\pi\Phi_{pp}(\omega) \tag{18}$$

The relations between the overall mean-square pressure and the the overall pressure level of the excitation are:

$$p_a^2 = 8.75526 \times 10^{-18}\ [10]^{L_a/10} \tag{19}$$

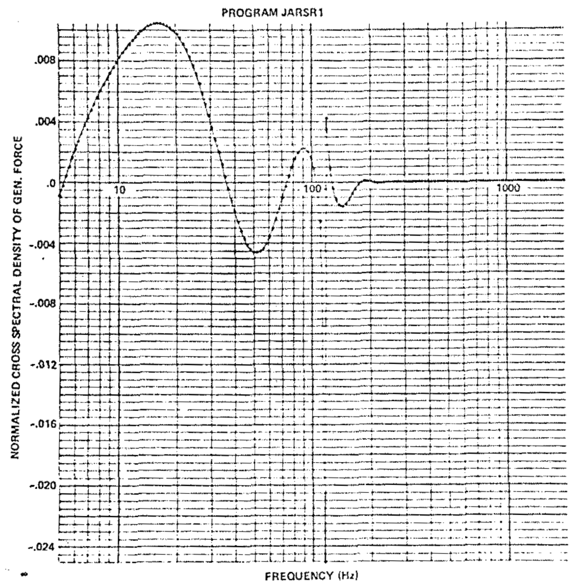


FIGURE 4. NORMALIZED CROSS SPECTRAL DENSITY OF GENERALIZED FORCE, SINUSOIDAL DEDECAYING CORRELATION FUNCTION, MODE INDICES: 1, 1, 1, 3

and

$$L_a = 170.576 + 10\log_{10}(p_a^2) \tag{20}$$

where

p_a^2 = mean-square pressure in $(\text{psi})^2$

L_a = excitation overall pressure level in decibels referenced 0.00002 Newton/meter^2

RESPONSES AT ANY POINT

The displacement response spectral density at any point $\vec{r}(x,y)$ is given by

$$\Phi_{ww}(\vec{r},\omega) = S'^2\Phi_{pp}(\omega) \sum_{j,k,m,n} F_{jk}F_{mn}|H_{jk}|\,|H^*_{mn}|J_{jkmn} \tag{21}$$

The magnitudes of the complex frequency response functions and their complex conjugates are given by

$$|H_{jk}| = M_{jk}^{-1}\left[(\omega^2_{jk} - \omega^2)^2 + (2\zeta_{jk}\omega_{jk}\omega)^2\right]^{-1/2} \tag{22}$$

The modal mass is given by

$$M_{jk} = \int MF^2{}_{jk}\, d\vec{r} \tag{23}$$

The mass distribution per unit area is:

$$M = \rho h + \rho' h' \tag{24}$$

The mean-square displacement is given by

$$w^2(\vec{r}) = \int \Phi_{ww}(r,\omega)d\omega \tag{25}$$

The root-mean-square displacement is given by the square root of $\overline{w^2}$.

The acceleration response spectral density is given by

$$\Phi_{\ddot{w}\ddot{w}}(\vec{r},\omega) = \omega^4 \Phi_{ww}(\vec{r},\omega) \tag{26}$$

When the acceleration spectral density is in in^2/sec^4/rad/sec, the acceleration response spectral density in g^2/Hertz is given by

$$S_{\ddot{w}\ddot{w}}(\vec{r},f) = 4.215093 \times 10^{-5}\, \Phi_{ww}(\vec{r},\omega) \tag{27}$$

The acceleration spectral density in decibels referenced "g" is

$$S_{\ddot{w}g}(\vec{r},f) = 10 \log_{10}\left[S_{\ddot{w}\ddot{w}}(\vec{r},f)\right] \tag{28}$$

The stress response spectral density is given by

$$\Phi_{\sigma\sigma}(\vec{r},\omega) = \gamma^2(\vec{r})\Phi_{ww}(\vec{r},\omega) \tag{29}$$

The factor to convert the displacement response into stress response is given by

$$\gamma^2(\vec{r}) = \frac{(Eh_1)^2}{4(1-\nu^2)} \cdot \frac{Q^2_x + Q^2_y}{Q^2_w} \tag{30}$$

where

$$Q_x = \sum_{\substack{m,n\\=1,3\,...}} \frac{\pi^2}{mn}\left[\left(\frac{m}{\ell}\right)^2 + \nu\left(\frac{n}{b}\right)^2\right] \cdot \frac{\sin\frac{m\pi x}{\ell}\sin\frac{n\pi y}{b}}{D_x\left(\frac{m}{\ell}\right)^4 + 2H\left(\frac{mn}{\ell b}\right)^2 + D_y\left(\frac{n}{b}\right)^4}$$

$$Q_y = \sum_{\substack{m,n\\=1,3,...}} \frac{\pi^2}{mn}\left[\left(\frac{n}{b}\right)^2 + \nu\left(\frac{m}{\ell}\right)^2\right] \cdot \frac{\sin\frac{m\pi x}{\ell}\sin\frac{n\pi y}{b}}{D_x\left(\frac{m}{\ell}\right)^4 + 2H\left(\frac{mn}{\ell b}\right)^2 + D_y\left(\frac{n}{b}\right)^4} \tag{31}$$

$$Q_w = \sum_{\substack{m,n\\=1,3,...}} \frac{\sin\frac{m\pi x}{\ell}\sin\frac{n\pi y}{b}}{mn\left[D_x\left(\frac{m}{\ell}\right)^4 + 2H\left(\frac{mn}{\ell b}\right)^2 + D_y\left(\frac{n}{b}\right)^4\right]} \tag{32}$$

AVERAGE RESPONSES

The average response over the whole structure is obtained by integrating the response over the structure and dividing the result with the area of the structure. Detail derivation of the formulas is given in [3]. The average displacement spectral density is

$$\Phi_{ww}(\omega) = S'^2\Phi_{pp}(\omega)\sum_{j,k} I_x I_y |H_{jk}|^2 J_{jkjk} \tag{33}$$

where the expressions for I_x and I_y for different boundary conditions are as follows:

Four Edges Simply-Supported Panels:

$$I_x = I_y = \frac{1}{2} \tag{34}$$

Four Edges Clamped Panels:

$$I_x(1) = I_y(1) = \frac{1}{1.5056\pi}\Big[1.5056\pi - 1 + \frac{1}{2}\sinh 2\,(1.5056\pi) - \sinh^2(1.5056\pi) - 2\exp(-1.5056\pi)\Big] \tag{35}$$

$$I_x = \frac{1}{(j+\tfrac{1}{2})\pi}\Big[(j+\tfrac{1}{2})\pi - 1 + \frac{1}{2}\sinh 2\,(j+\tfrac{1}{2})\pi - \sinh^2(j+\tfrac{1}{2})\pi - 2(-1)^{j+1}\exp\{-(j+\tfrac{1}{2})\pi\}\Big]$$

$$j = 2, 3, \ldots \tag{36}$$

$I_y(k)$ is given by eq. (36) with j replaced by k.

Two opposite edges simply-supported while other two clamped:

$$I_x = \frac{1}{2} \tag{37}$$

$I_y(k)$ is given by eqs. (35) and (36) with j replaced by k.

ONE-NTH OCTAVE FREQUENCY INCREMENT

In the calculation of the response spectrum as a function of frequency, uniform frequency increment is not convenient, because when the spectrum is plotted with the frequency in logarithmic scale, the points will be too close in high frequency and too separated in low frequency. In order to obtain good plots and save computer time, it is convenient to use one-nth-octave frequency increment.

The interval from frequency f_1 to f_2 will be one-nth octave if

$$\left(\frac{f_2}{f_1}\right)^n = 2 \tag{38}$$

The geometric mean frequency of f_1 and f_2 is

$$f = (f_1 f_2)^{1/2} \tag{39}$$

It can be shown that the bandwidth of the one-nth octave band is

$$f_2 - f_1 = D_n f = D'_n f_1 \tag{40}$$

where the one-nth-octave bandwidth constants are given by

$$D_n = 2^{\frac{1}{2n}} - 2^{-\frac{1}{2n}} \tag{41}$$

$$D'_n = 2^{\frac{1}{n}} - 1 \tag{42}$$

Some values of D'_n and D_n for n from 1 to 50 are as follows. Note that when n equals to 1, 2, and 3, the values of D'_n and D_n are the corresponding one-octave, one-half-octave and one-third-octave bandwidth constants.

Value of n	Value of D′	Value of D
1	1	0.707107
2	0.41421	0.348311
3	0.25992	0.231563
10	0.07177	0.0693286
20	0.03526	0.0346591
30	0.02337	0.0231054
40	0.01748	0.0173289
50	0.01359	0.0138631

When the one-nth-octave bandwidth is used for frequency increments, the expression for the number of data points will be

$$i = n \frac{\log \frac{f_j}{f_o}}{\log 2} + 1 \qquad (43)$$

where f_o and f_j are the initial and final frequencies respectively. For example

$$n = 33$$

$$f_o = 5$$

$$f_j = 5000$$

Substituting into eq. (43) gives the number of data points

$$i = 331$$

VIBRO-ACOUSTIC TRANSFER FUNCTION

The vibro-acoustic transfer function at any point of a structure is defined as the ratio of the response to the excitation. When the response and the excitation are expressed in decibels, the vibro-acoustic transfer function in decibels at any point $\vec{r}$ will be the excitation in decibels minus the response in decibels:

$$T_{\ddot{W}}(\vec{r},f) = S_{\ddot{W}g}(\vec{r},f) - S'_{pp}(f) \qquad (44)$$

$T_{\ddot{W}}(\vec{r},f)$ = Vibro-acoustic transfer function in decibels with excitation pressure referenced 0.00002 Newton/meter^2 and acceleration response referenced g

$S_{\ddot{W}g}(\vec{r},f)$ = Acceleration spectral density in decibels referenced g given by eq. (28)

$S'_{pp}(f)$ = The spatial homogeneous excitation spectral density in psi^2/Hz given by eq. (18)

Program RSRPC2 is written to investigate the vibro-acoustic transfer function of four edges simply-supported rectangular cylindrical shell panels cross reinforced with ribs and stringers. Typical plots of this program are shown in Figures 5 and 6.

COMPUTER PROGRAMS AND RESULTS

1. Program RANDOM

Program RANDOM [4] is a combination of the three programs RSRPC1, RFRPC1, and RSFRP1. With this program a single loading of the input data will be sufficient to obtain the responses of a rectangular cylindrical shell panel cross-reinforced with stiffeners and subjected to three boundary conditions: all edges simply-supported, all edges clamped, and

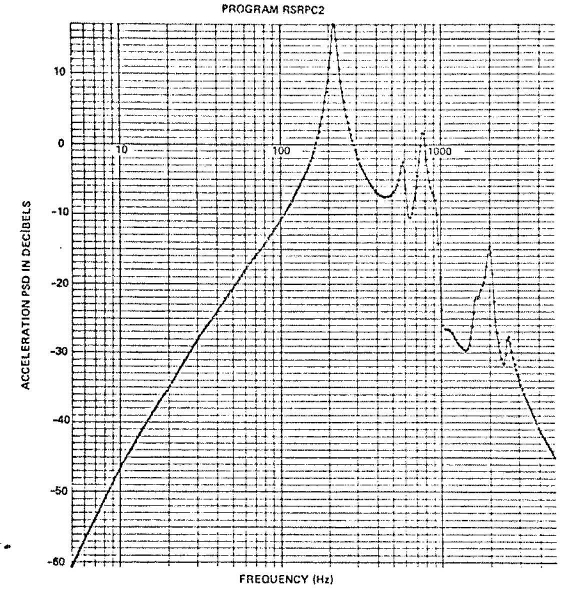


FIGURE 5. DECIBEL SCALE ACCELERATION SPECTRAL DENSITY AT CENTER OF SIMPLY-SUPPORTED CURVED RECTANGULAR PANEL CROSS-REINFORCED WITH STIFFENERS

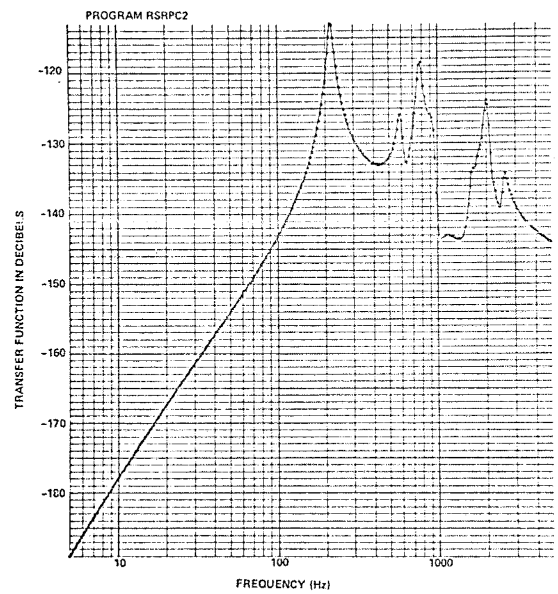


FIGURE 6. VIBRO-ACOUSTIC TRANSFER FUNCTION AT CENTER OF SIMPLY-SUPPORTED CURVED RECTANGULAR PANEL CROSS-REINFORCED WITH STIFFENERS

two opposite edges simply-supported with other two clamped. Controls are provided to run any one of the boundary conditions.

Input data for this program are (a) the material constants, the geometric dimensions and properties of the shell panel and stiffeners, (b) the one-third-octave spectrum of the excitation pressure, (c) x and y coordinates of the point of interest, and (d) some control constants.

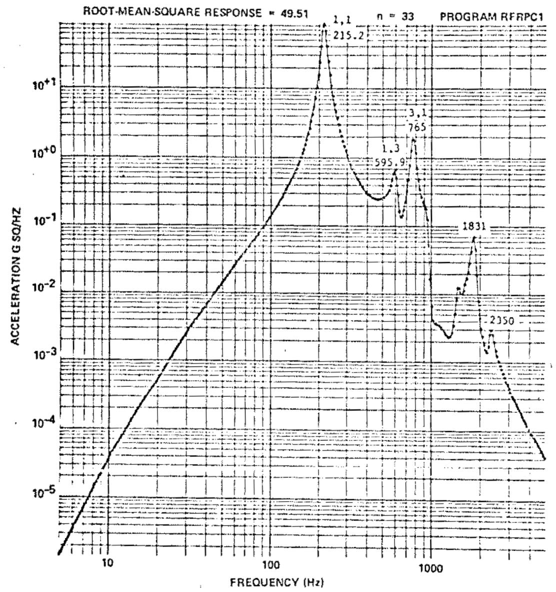
The output data of this program include:

a. All the input data with nomenclature.

b. The input one-third-octave spectrum of the excitation pressure, this excitation spectrum is converted into spectral density both in decibel/Hertz and $(psi)^2$/Hertz, tabulated and plotted.

c. Natural frequencies of the panel both in Hertz and radian/second.

d. Spectral densities of displacement in $(inch)^2$/Hertz, of stress in $(psi)^2$/Hertz, of acceleration in g^2/Hertz and the excitation spectral density in $(psi)^2$/rad/sec. In addition to tabulation, all three spectral densities are plotted. Typical plots are shown in Figures 7, 8, and 9.



NOTE: NUMBERS AT PEAKS INDICATE MODE INDICES AND FREQUENCIES.

FIGURE 7. ACCELERATION SPECTRAL DENSITY AT CENTER OF FOUR-EDGES CLAMPED CURVED RECTANGULAR PANEL CROSS-REINFORCED WITH STIFFENER

e. The mean-square and the root-mean-square values of responses and excitation.

f. Constants Q_x, Q_y, and Q_w and the constant γ^2 to convert the displacement spectral density into stress spectral density.

g. Some values of the joint acceptance square for all combinations of modes.

2. Programs RSRPC1, RSRPC2, RSRPC3, and RSRPC4

These four programs are written to calculate the responses of the simply-supported cylindrical shell rectangular panel cross-reinforced with ribs and stringers for various specific purposes. Input data for these programs are the same as Program RANDOM. The output data for these four programs are the same as Program RANDOM with extra output for each individual program.

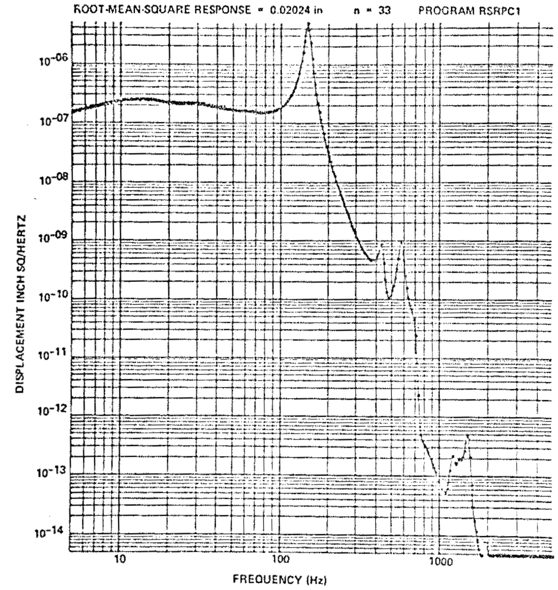


FIGURE 8. DISPLACEMENT SPECTRAL DENSITY AT CENTER OF FOUR EDGES SIMPLY-SUPPORTED CURVED RECTANGULAR PANEL CROSS-REINFORCED WITH STIFFENERS

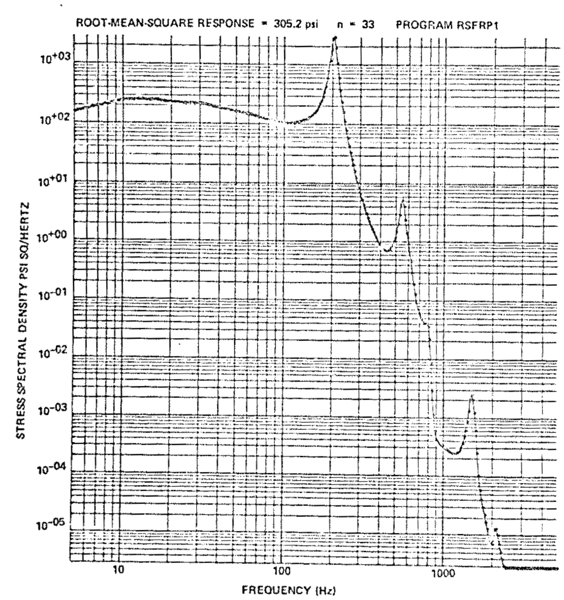


FIGURE 9. STRESS SPECTRAL DENSITY AT CENTER OF TWO OPPOSITE EDGE SIMPLY-SUPPORTED AND OTHER TWO CLAMPED RECTANGULAR CURVED PANEL CROSS-REINFORCED WITH STIFFENERS

Program RSRPC1 calculates the spectral densities of the displacement response in $(inch)^2$/Hertz, the stress response in $(psi)^2$/Hertz, and the acceleration response in g^2/Hertz. Therefore the output data are identical with Program RANDOM run in the case of all edges simply-supported.

In addition to those as in Program RSRPC1, the output data of Program RSRPC2 include the acceleration spectral density in decibels referenced gravity acceleration and the vibro-acoustic transfer function as a function of frequency. Both the transfer function and the decibel scale acceleration spectral density are plotted. (See Figures 5 and 6.)

Program RSRPC3 is designed to investigate the contribution of the cross terms to the responses. Both the responses of all terms summation and cross terms neglected are tabulated and plotted for comparison. Typical plots of this program are shown in Figures 10 and 11.

Program RSRPC4 is a modification of Program RSRPC1 to calculate responses at any point of the structure and the average response over the whole structure. Typical plot of this program is shown in Figure 12.

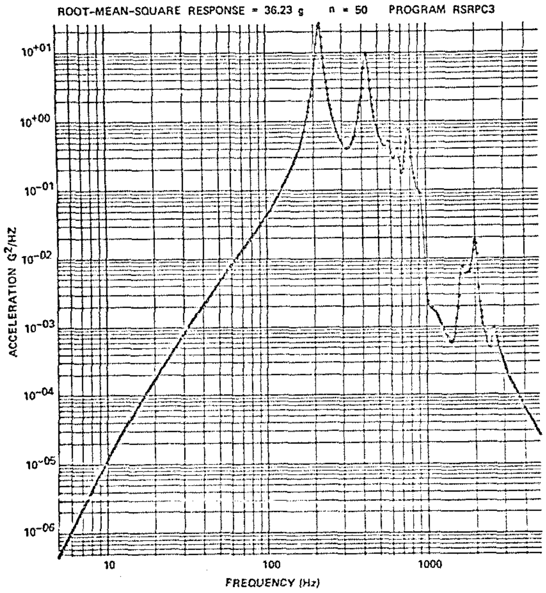


FIGURE 10. ACCELERATION SPECTRAL DENSITY, ALL TERMS SUMMATION, AT x = ℓ/4 = 11.87 in., y = b/2 = 29.187 in.

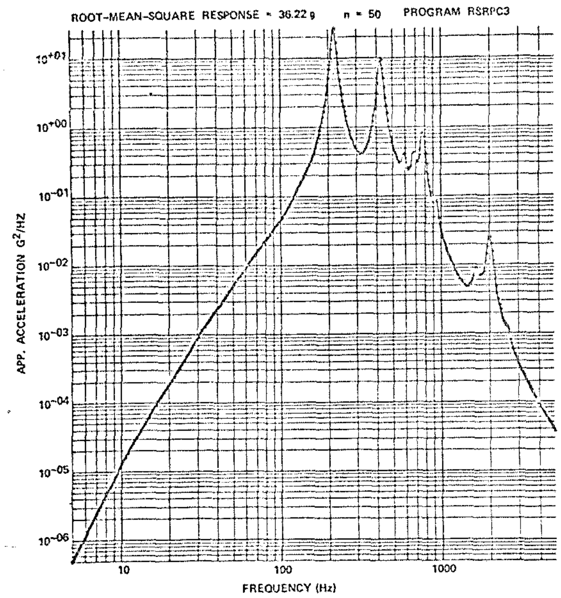


FIGURE 11. ACCELERATION SPECTRAL DENSITY, CROSS TERMS NEGLECTED, AT x = ℓ/4 = 11.87 in., y = b/2 = 29.187 in.

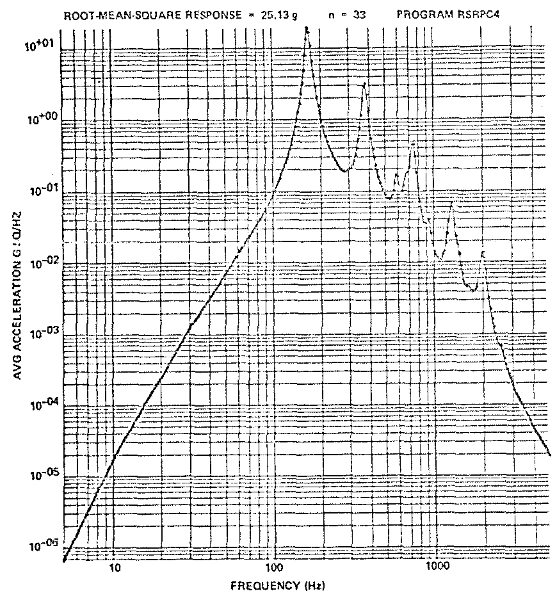


FIGURE 12. AVERAGE ACCELERATION SPECTRAL DENSITY OVER WHOLE PANEL, RADIUS OF CURVATURE a = 100 INCHES

3. Program RFRPC1 and RFRPC4

These programs are written to calculate the vibrational responses of the four edges clamped cylindrical shell rectangular panel cross-reinforced with ribs and stringers. The input data for these programs are the same as Program RANDOM. The output data for these programs are essentially the same as Program RANDOM except Program RFRPC4 also gives average responses over the whole panel.

Program RFRPC1 calculates and plots spectral densities of the displacement response in $(\text{inch})^2$/Hertz, the stress response in $(\text{psi})^2$/Hertz, and the acceleration response in g^2/Hertz. Therefore, the output data are identical with Program RANDOM run in the case of all edges clamped, except the frequency range can be higher and the frequency increment can be smaller.

Program RFRPC4 is a modification of Program RFRPC1. In addition to the output of Program RFRPC1, Program RFRPC4 calculates the average responses over the whole structure.

4. Programs RSFRP1 and RSFRP4

These programs are written to calculate the vibrational response of the two opposite edges simply-supported while other two clamped rectangular cylindrical shell panel cross-reinforced with ribs and stringers. The input data for these programs are the same as Program RANDOM. The output data for these programs are the same as Program RANDOM except that Program RSFRP4 also calculates the average responses over the whole panel.

Program RSFRP1 calculates and plots the spectral densities of the displacement response in $(inch)^2$/Hertz, the stress response in $(psi)^2$/Hertz, and the acceleration response in g^2/Hertz. Therefore, the output data are identical with Program RANDOM run in the case of two opposite edges simply-supported while other two clamped, except with higher frequency range and smaller frequency increment.

Program RSFRP4 is a modification of RSFRP1. In addition to calculating local responses at any point of the structure, Program RSFRP4 also calculates and plots average responses over the whole panel.

5. Program JARSR1

This is a computer program to study the correlation coefficient as a function of separation distance, the joint acceptance and the normalized cross spectral density of the generalized force as a function of frequency for various correlation functions. Typical plots of the joint acceptance and the normalized cross spectral density of the generalized force are shown in Figures 2, 3 and 4.

6. Program NFUOP1

Program NFUOP1 is to calculate the total number of modes and the modal density. The input data to this program are the material constants, the geometric dimensions and properties of the structure required in the frequency equations. The output data of this program are the tabulated and plotted number of modes and the modal density as a function of frequency. Sample results are given in Figures 13 and 14. The analysis of the number of modes and modal density is useful in the evaluation of the frequency equations. It enables the structure engineer to determine the accuracy and the behavior of the frequency equations. This program can be modified to apply to any structure.

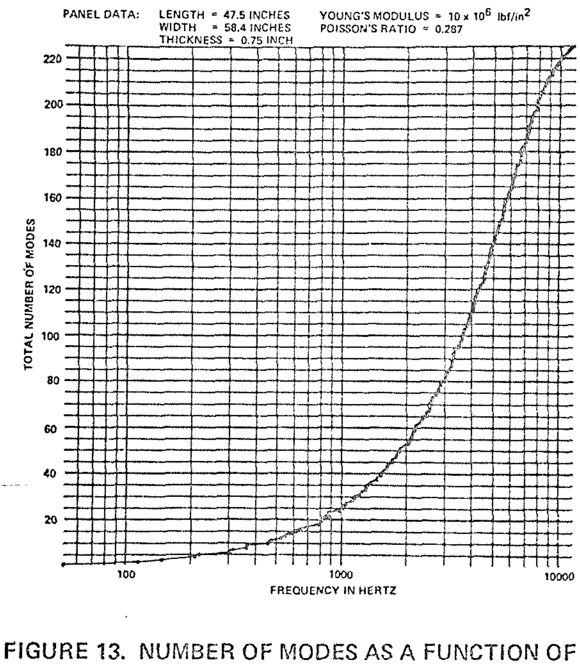


FIGURE 13. NUMBER OF MODES AS A FUNCTION OF FREQUENCY OF A SIMPLY-SUPPORTED UNIFORM RECTANGULAR PANEL

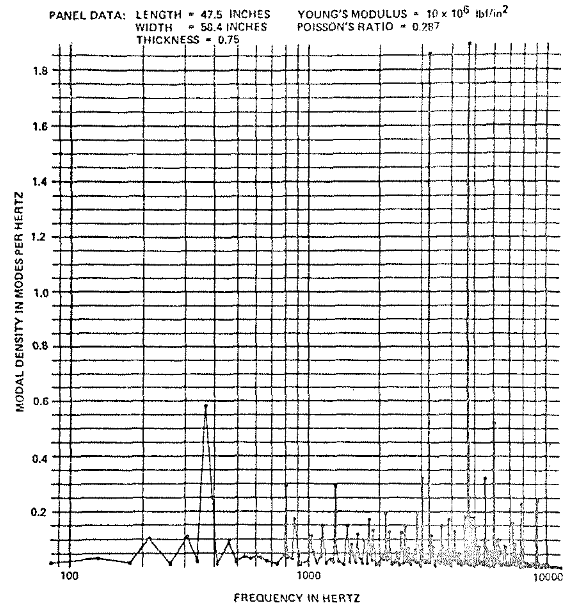


FIGURE 14. MODAL DENSITY OF A SIMPLY-SUPPORTED UNIFORM RECTANGULAR PANEL

COMPARISON OF COMPUTED RESULTS WITH TEST DATA

The developed computer programs have been used in the research project which Chrysler Huntsville Operations conducted for Marshall Space Flight Center under Contract No. NAS8-21425. The object of this project is to develop comparative analysis of acoustic testing techniques and to determine the time for acoustic qualification test at levels other than specified level. Comparison of computed results with test data shows good agreement.

One of the test specimens in Project NAS8-21425 was a 4 in. x 13 in., 0.2 in. thick aluminum flat plate. The plate was knife-edged supported on four sides and was subjected to high level acoustic pressure until fatigue failure occurred. The input spectrum of the acoustic pressure is shown in Figure 15. Figure 16 shows the calculated acceleration spectral density compared with test data at location 9, the coordinates of which are $x = 6.5$ inches and $y = 1$ inch.

CONCLUSIONS

1. In the calculation of random structural vibrational responses due to the fluctuating pressure environments by the normal mode approach, the important factors that have to be determined are:

a. The normal modes

b. The natural frequencies

c. The joint acceptance or the cross spectral density of the generalized force which in turn depends on:

(1) The normal modes

(2) The correlation properties of the pressure field

(3) The spectral density of the excitation pressure.

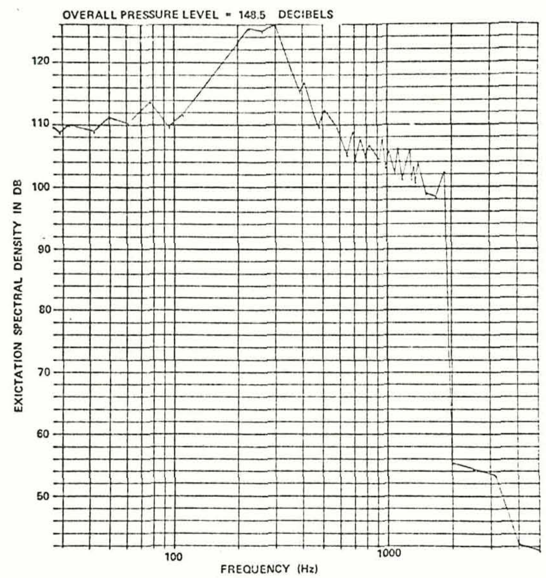


FIGURE 15. ACOUSTIC PRESSURE SPECTRAL DENSITY IN DECIBELS FOR FATIGUE TEST IN PROJECT NAS8-21425

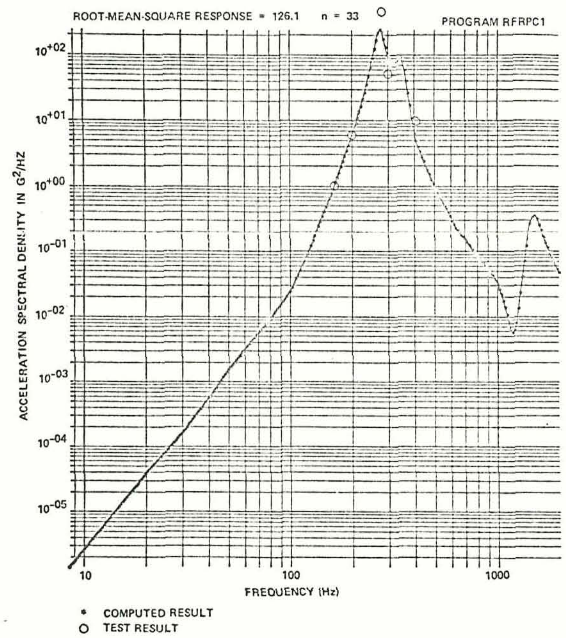


FIGURE 16. COMPARISON OF CALCULATED RESULT WITH TEST RESULT, LOCATION NO. 9

2. For simple structures, the determination of the mode shapes and frequencies can be obtained to a certain degree of accuracy. For complex structures, the mode shapes and frequencies are difficult to determine in general.

3. For simple pressure fields, the correlation properties can be obtained analytically. The correlation properties of complex pressure fields will depend on experimental data.

4. Any discrepancy of the factors mentioned above will affect the computed results.

5. To develop computer programs for the calculation of these random vibrations, the task is to search or develop the necessary frequency equations, the normal modes, the analytical expressions for the correlation properties and the joint acceptance. The accuracy of these quantities should be consistent. When these quantities are incorporated in the formulations of the programs, the required computer time should be moderate as to make them practical for use. Such task has been done in this project.

6. One of the important features of the computer programs developed here is that any shape of the spectrum of the excitation pressure can be input into the programs. Thus, the simulation of the excitation will be as accurate as the spectral analysis of the random pressure.

7. The discrepancy in the determination of the natural frequencies will affect the response spectrum to some degree, while it will not affect the overall mean-square response very much.

8. It is well known that the determination of the damping properties of structures is very difficult. By utilization of the developed computer programs and the results of test data, the damping properties of structures can be determined. This is another application of these programs.

ACKNOWLEDGEMENTS

This is the accomplishment of a Marshall Space Flight Center Contract. The authors wish to express their appreciation to Dr. Hugo Steiner of the Marshall Space Flight Center for his guidance and assistance. The authors also wish to thank Mr. James Kermit Moore of Chrysler for writing the computer programs.

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DISCUSSION

Mr. Lyon (Bolt Beranek & Newman): What happens to the value of response of your test structures as the internal damping goes to zero?

Mr. Lee: We have not tried it. I used four percent damping to obtain my results.

Mr. Lyon: One of the major results of all the work that has gone on in sound-structure interaction in the last ten years has shown that there is a finite upper limit to the response, even in the absence of internal damping which is associated with an energy sharing between the sound field and the structure. Therefore, since radiation damping in some of these structures can be an important part of the total damping, I would question whether or not one can infer the internal damping from a comparison of observed response with the calculations.

Mr. Lee: I think the damping here simply included the internal damping as a part of the radiation damping. That is why we have to use four percent or six percent damping. Ordinarily, structural damping is less.